NEW TRENDS IN CALCULUS OF VARIATIONS AND CONTINUUM MECHANICS

ON THE OCCASION OF THE 70TH BIRTHDAY OF DANILO PERCIVALE

Room 706, Department of Mathematics, University of Genoa
`https://cvcm.unige.it/`

Program

Thursday 11 June

10:30 Registration & Opening

11:00 **Giuseppe Buttazzo** (University of Pisa):
Optimal data for elliptic PDEs

11:45 **Gianni Dal Maso** (SISSA, Trieste):
Gamma-Convergence and stochastic homogenization for functionals in the A-free setting

12:30 Lunch

14:00 **Emilio Acerbi** (University of Parma):
Bifurcation for a Sharp Interface Generation Problem

14:45 **Matteo Negri** (University of Pavia):
Phase-field models for fracture: recent developments and perspectives

15:30 Coffee break

16:00 **Luigi Ambrosio** (Scuola Normale Superiore, Pisa):
A survey on the theory of Regular Lagrangian flows

Friday 12 June

9:00 **Ulisse Stefanelli** (University of Vienna):
Phase-field models for accretive growth in viscoelastic solids

9:45 **Flaviana Iurlano** (University of Genoa):
Quasi-static elastic contact with Coulomb friction: transition from continuous to jumping solutions in 2D

10:30 Coffe Break

11:00 **Simone Di Marino** (University of Genoa):
An Optimal Tranport approach to Equilibrium in Market Areas

11:45 **Martin Kružík** (Czech Academy of Sciences, Prague):
On the passage from nonlinear to linearized viscoelastodynamics

12:30 **Franco Tomarelli** (Istituto Lombardo Accademia di Scienze e Lettere & Politecnico di Milano):
Symmetrized Fractional Variation for signal denoising

13:15 Lunch

List of Abstracts

Bifurcation for a Sharp Interface Generation Problem

Emilio Acerbi
University of Parma

As opposed to the widely studied bifurcation phenomena for maps or PDE problems, we are concerned with bifurcation for stationary points of a non-local variational functional defined not on functions but on sets of finite perimeter. We consider the sharp interface limit of the FitzHugh-Nagumo energy functional in a flat torus in R^2, for which lamellar stationary points exist and their stability range is well understood, and we prove that when the lamella loses its stability bifurcation occurs, leading to a two-dimensional branch of nonplanar stationary points. The same results are obtained for multi-lamellae, which exhibit a transition from supercritical to subcritical bifurcation as the torus size increases. To the best of our knowledge, bifurcation for nonlocal problems in a geometric measure theoretic setting is an entirely new result

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A survey on the theory of Regular Lagrangian flows

Luigi Ambrosio
Scuola Normale Superiore, Pisa

The theory of Regular Lagrangian Flows opens the possibility to establish well posedness and stability with respect to perturbations for several classes of ordinary differential equations associated to nonsmooth vector fields, even in the absence of noise, with applications to fluid mechanics and conservation laws. Initiated by papers of Di Perna and Lions and Ambrosio (who dealt with the class of Sobolev vector fields and BV vector fields) this theory has by now reached a mature stage. In the Colloquium I will survey about old and recent developments, including fine properties of the Regular Lagrangian Flow and even the case of nonsmooth velocity fields in nonsmooth environments.

Optimal data for elliptic PDEs

Giuseppe Buttazzo
University of Pisa

The goal is to present some optimization problems related to elliptic PDEs; the governing equations have Dirichlet boundary conditions on a given domain Ω and are of the form

$$-\mathrm{div}(a(x)\nabla u) = f \quad \text{where } a(x) \text{ is the control variable;}$$
$$-\Delta u + V(x)u = f \quad \text{where } V(x) \text{ is the control variable;}$$
$$-\Delta u = f \quad \text{where } f(x) \text{ is the control variable.}$$

We will mainly focus on the first case, where the coefficient $a(x)$ has to be determined, in a suitable given class of admissible choices, in order to optimize a given criterion. We first deal with the case when the cost is the so-called elastic compliance, and then we discuss the more general case when the problem is written as an optimal control problem. The case of elasticity problems, where u is vactor-valued, will be also presented.

Gamma-Convergence and stochastic homogenization for functionals in the A-free setting

Gianni Dal Maso
Scuola Internazionale Superiore di Studi Avanzati, Trieste

We obtain a compactness result for Gamma-convergence of integral functionals defined on A-free vector fields. This is used to study homogenization problems for these functionals without periodicity assumptions. More precisely, we prove that the homogenized integrand can be obtained by taking limits of minimum values of suitable minimization problems on large cubes, when the side length of these cubes tends to infinity, assuming that these limit values do not depend on the center of the cube. Under the usual stochastic periodicity assumptions, this result is then used to solve the stochastic homogenization problem by means of the subadditive ergodic theorem.

An Optimal Transport Approach to Equilibrium in Market Areas

Simone Di Marino
University of Genoa

In this ongoing study with Sara Farinelli, we want to explore equilibrium configurations in a Market Area problem. The setting studied is introduced in a recent paper by Santacesaria and Lanzara: farmers are transporting goods to cities with an ice melting discounted price, and the goods are bought (by both farmers and cities) according to the maximization of a CES utility function. Then the equilibrium is established at the level of production and consumption. We prove that the general problem of equilibrium can be recast as the maximization of a dual problem on prices, which can be seen as the dual of an optimal transport-entropy problem and we generalize the result to any positive 1-homogeneous utility function. We then employ the OT characterization to cast the problem of best location for the cities and their influence region.

Quasi-static elastic contact with Coulomb friction: transition from continuous to jumping solutions in 2D

Flaviana Iurlano
University of Genoa

We formulate the quasi-static elastic contact problem with Coulomb friction in a very general setting, with possible jumps in time for both the load and the solution. We exhibit an optimal condition on the magnitude of the friction coefficient under which we prove the existence of an absolutely continuous solution for arbitrary absolutely continuous loads in the case of the most general 2D problem. We provide examples showing that, when the condition is violated, spontaneous jumps in time of the solution may occur, even when the load varies absolutely continuously in time. We argue that these spontaneous jumps in time of the solution in the quasi-static problem reveal a transition of the process from a quasi-static nature to a dynamic nature.

On the passage from nonlinear to linearized viscoelastodynamics

Martin Kružík
Institute of Information Theory and Automation, Czech Academy of Sciences, Prague

The equations of linearized viscoelastodynamics in Kelvin-Voigt rheology are rigorously derived from a nonlinear model that satisfies the time-dependent frame indifference in the sense of Antman. Besides showing the convergence of corresponding solutions of both systems, we also prove the convergence of time-discrete solutions on various scales and of continuous solutions of nonlinear problems to linearized ones. This is a joint work with B. Benesova and M. Kampschulte.

Phase-field models for fracture: recent developments and perspectives

Matteo Negri
University of Pavia

The presentation provides an overview of recent theoretical developments, advanced modeling, and future perspectives in phase-field approximations for fracture mechanics. Rooted in the variational approach to free discontinuity problems and Γ-convergence, the talk addresses energy-splitting techniques designed to account for tension-compression asymmetry and non-interpenetration constraints. The presentation describes the characterization of quasi-static crack evolutions via time-discretization schemes, separate minimizers, and energy descent algorithms, highlighting the technical ingredients needed to gain a clear description of time-continuous evolutions. Finally, it explores a couple of recent extensions: the first to cohesive fracture, using elasto-plastic gradient-damage formulations, the second to tension-compression asymmetry, using a suitable variation of the AT_1 functional, in the framework of generalized standard materials. Both this new directions are supported by numerical simulations.

Phase-field models for accretive growth in viscoelastic solids

Ulisse Stefanelli
University of Vienna

Accretive growth is a fundamental mechanism in biological, natural, and technological systems. I will present two phase-field models for finite-strain viscoelastic solids undergoing mechanically driven accretion, addressing both phase-transition dynamics and the build-up of growth-induced incompatibilities.

The first model describes a two-phase viscoelastic medium in which one phase irreversibly accretes at the expense of another, with growth governed by a generalized eikonal equation coupled to quasistatic viscoelastic equilibrium. Both diffuse- and sharp-interface formulations are considered, and existence of weak/viscosity solutions is established together with energy balances and sharp-interface convergence.

The second model includes the description of incompatible growth through a multiplicative decomposition of deformation into elastic strain and backstrain, modeling the residual stresses generated as newly accreted material is deposited in an unstressed state. A compliant fictitious surrounding medium regularizes the free-boundary problem, enabling the proof of existence for the fully coupled system.

This is joint work with Andrea Chiesa (U. Vienna).

Symmetrized Fractional Variation for signal denoising

Franco Tomarelli
Istituto Lombardo Accademia di Scienze e Lettere
Politecnico di Milano

We define, study and implement the model SFV (Symmetrized Fractional Variation): a variational approach to signal analysis exploiting the Riemann-Liouville fractional derivatives of every positive real order higher than zero. The model exploits an L^1 fitting data term together with both right and left fractional derivatives as regularizing terms: this approach aims to achieve an orientation-independent protocol.
We express the model as an energy minimization and introduce a functional framework where existence of minimizers is proved: namely the space BV^s_*.
We show that the embedding in BV^s_* of the Sobolev space of the same fractional order is strict, and we exhibit some nontrivial borderline examples of admissible or non admissible functions in the space BV^s_*.
To provide evidence of effectiveness for the proposed model, a discretisation based on a second-order consistent Grünwald-Letnikov scheme is introduced.
A multi-parameter whiteness criterion is proposed which provides an unsupervised, automatic and simultaneous selection of the two free parameters in the model, namely the fractional order of differentiation and the regularization parameter.
Numerical experiments on 1-d and 2-d signals are performed which show how the proposed model holds the potential to achieve good quality results for denoising signals corrupted by additive Laplace noise.

This is a joint research with Alessandro Lanza(1), Antonio Leaci(2) and Serena Morigi(1).

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